NAG Toolbox for MATLAB

s22aa

1 Purpose

s22aa returns a sequence of values for either the unnormalized or normalized Legendre functions of the first kind $P_n^m(x)$ or $\overline{P_n^m}(x)$ for real x of a given order m and degree $n = 0, 1, \ldots, N$.

2 Syntax

```
[p, ifail] = s22aa(mode, x, m, nl)
```

3 Description

s22aa evaluates a sequence of values for either the unnormalized or normalized Legendre ($m = 0$) or associated Legendre ($m \neq 0$) functions of the first kind $P_n^m(x)$ or $\overline{P_n^m}(x)$, where x is real with $-1 \leq x \leq 1$, of order m and degree $n = 0, 1, \ldots, N$ defined by

$$\begin{aligned} P_n^m(x) &= \left(1 - x^2\right)^{m/2} \frac{d^m}{dx^m} P_n(x) && \text{if } m \geq 0, \\ P_n^m(x) &= \frac{(n+m)!}{(n-m)!} P_n^{-m}(x) && \text{if } m < 0 \quad \text{and} \\ \overline{P_n^m}(x) &= \sqrt{\frac{(2n+1)}{2} \frac{(n-m)!}{(n+m)!}} P_n^m(x) \end{aligned}$$

respectively; $P_n(x)$ is the (unassociated) Legendre polynomial of degree n given by

$$P_n(x) \equiv P_n^0(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} \left(x^2 - 1\right)^n$$

(the *Rodrigues formula*). Note that some authors (e.g., Abramowitz and Stegun 1972) include an additional factor of $(-1)^m$ (the *Condon–Shortley Phase*) in the definitions of $P_n^m(x)$ and $\overline{P_n^m}(x)$. They use the notation $P_{mn}(x) \equiv (-1)^m P_n^m(x)$ in order to distinguish between the two cases.

s22aa is based on a standard recurrence relation described in Section 8.5.3 of Abramowitz and Stegun 1972. Constraints are placed on the values of m and n in order to avoid the possibility of machine overflow. It also sets the appropriate elements of the array **p** (see Section 5) to zero whenever the required function is not defined for certain values of m and n (e.g., $m = -5$ and $n = 3$).

4 References

Abramowitz M and Stegun I A 1972 *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Parameters

5.1 Compulsory Input Parameters

1: **mode – int32 scalar**

Indicates whether the sequence of function values is to be returned unnormalized or normalized.

mode $= 1$

The sequence of function values is returned unnormalized.

mode $= 2$

The sequence of function values is returned normalized.

Constraint: **mode** $= 1$ or 2.

2: **x – double scalar**

The argument x of the function.

Constraint: $\text{abs}(\mathbf{x}) \leq 1.0$.

3: **m – int32 scalar**

The order m of the function.

Constraint: $\text{abs}(\mathbf{m}) \leq 27$.

4: **nl – int32 scalar**

The degree N of the last function required in the sequence.

Constraints:

$\mathbf{nl} \geq 0$;
if $\mathbf{m} = 0$, $\mathbf{nl} \leq 100$;
if $\mathbf{m} \neq 0$, $\mathbf{nl} \leq 55 - \text{abs}(\mathbf{m})$.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **p(0 : nl) – double array**

The required sequence of function values as follows:

if **mode** $= 1$, $\mathbf{p}(n)$ contains $P_n^m(x)$, for $n = 0, 1, \ldots, N$;
if **mode** $= 2$, $\mathbf{p}(n)$ contains $\overline{P_n^m}(x)$, for $n = 0, 1, \ldots, N$.

2: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

On entry, $\text{abs}(\mathbf{x}) > 1.0$,
or **mode** $\neq 1$ or 2,
or **nl** < 0,
or **nl** > 100 when **m** $= 0$,
or $\text{abs}(\mathbf{m}) > 27$,
or **nl** $+ \text{abs}(\mathbf{m}) > 55$ when **m** $\neq 0$.

7 Accuracy

The computed function values should be accurate to within a small multiple of the ***machine precision*** except when underflow (or overflow) occurs, in which case the true function values are within a small multiple of the underflow (or overflow) threshold of the machine.

8 Further Comments

None.

9 Example

```
mode = int32(1);
x = 0.5;
m = int32(2);
nl = int32(3);
[p, ifail] = s22aa(mode, x, m, nl)
```

```
p =
         0
         0
    2.2500
    5.6250
ifail =
           0
```